

Cascading Failures on Power Grids

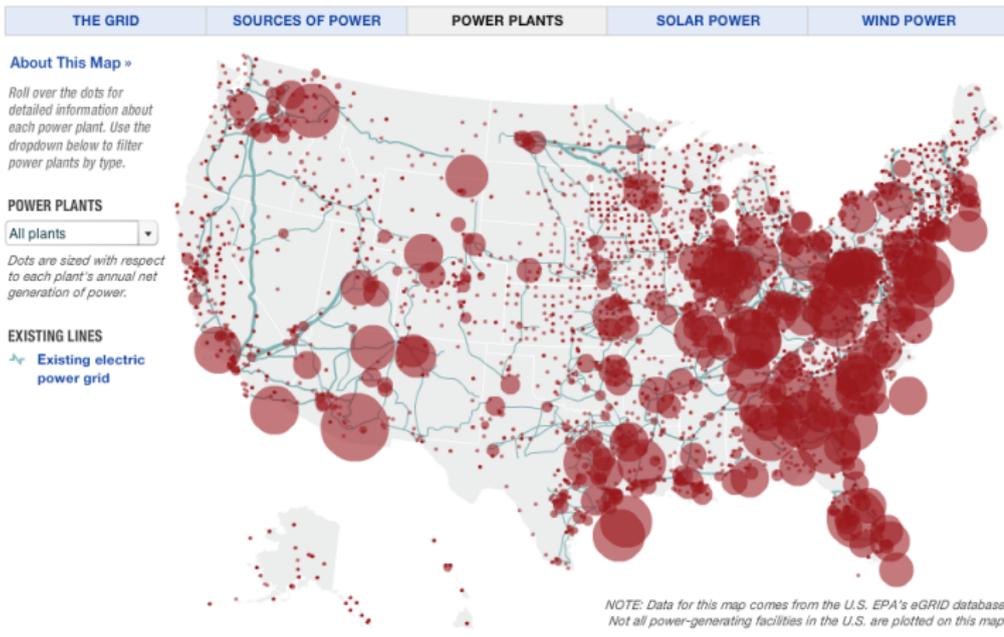
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Including some joint work with Sachin Kadloor (UIUC)

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Outline

- 1 Background
- 2 Prior Work
- 3 Initial System Model
- 4 A Simple Failure Model
- 5 A Perturbation Analysis
- 6 Improvements to the System Model
- 7 Available Transmission Grid Data
- 8 Visualizing the Grid
- 9 Conclusions

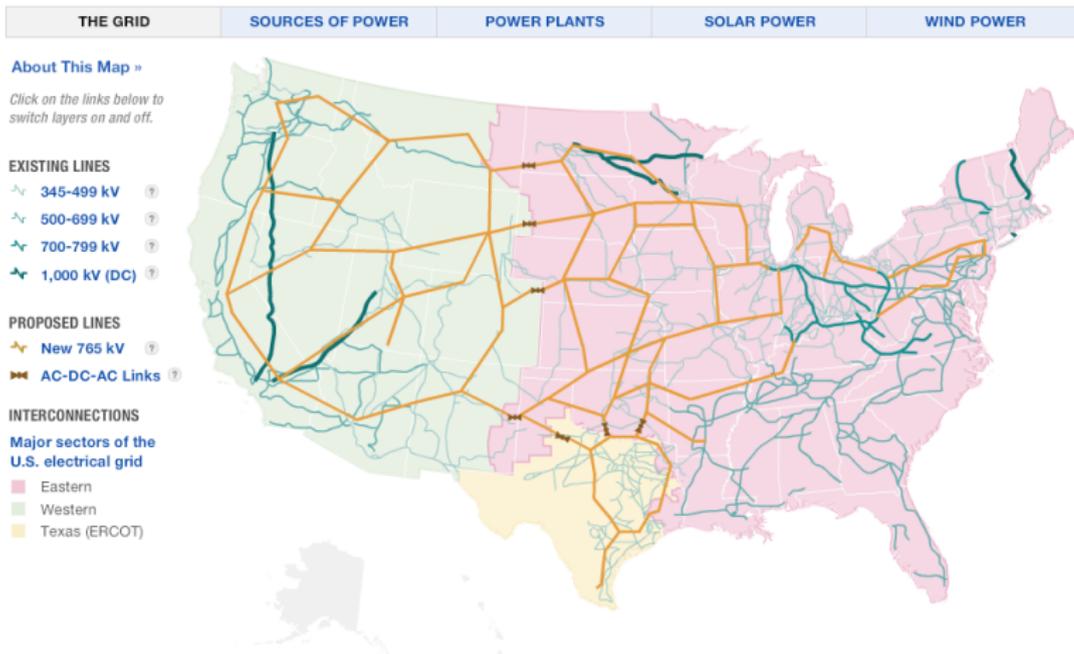
Background: US Electric Power Plants



Data: U.S. EPA's eGrid database

Visualization: www.npr.org/templates/story/story.php?storyId=110997398

Background: US Electric Transmission Grid



Data: Various sources

Visualization: www.npr.org/templates/story/story.php?storyId=110997398

- Power Grid has large number of interacting components
- Several failure mechanisms: hidden failures, operator error, shorting of lines due to lack of maintenance, relays misbehaving due to over-maintenance, erratic consumer demands, lightning, earthquakes etc
- Non local effects of failures

- Large blackouts are typically triggered by very few (one or two) primary events, which are followed by a cascading sequence of secondary failures
- Larger disruptions are less probable: probability is a decreasing power function of event size
- Larger blackouts though rarer, are much more costlier
- So it pays to study cascading failures

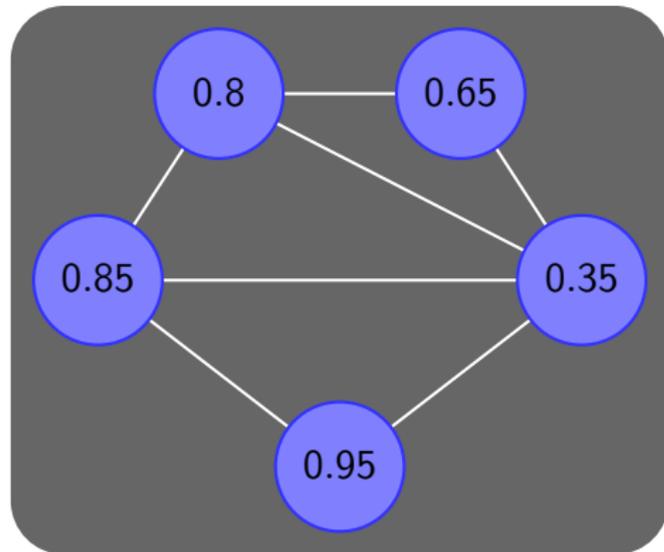
Grid Specific

- Each failing node increases the load on every other node uniformly (Dobson, 2004)
- Branching process: each failing node takes with it a random number of nodes (Dobson, 2004)

General Networks

- A node fails if a fraction of its neighbors fail (Watts, 2002)
- Drop in efficiency of a network because of an imbalance in flow distribution (Crucitti et al., 2004; Latora et al., 2001)

A Simple Initial System Model



High level abstraction of the grid

- Grid is modeled as an undirected graph
- Nodes are generators and weights are loads served
- All nodes have unit capacity
- Generator is online if load demand is less than capacity

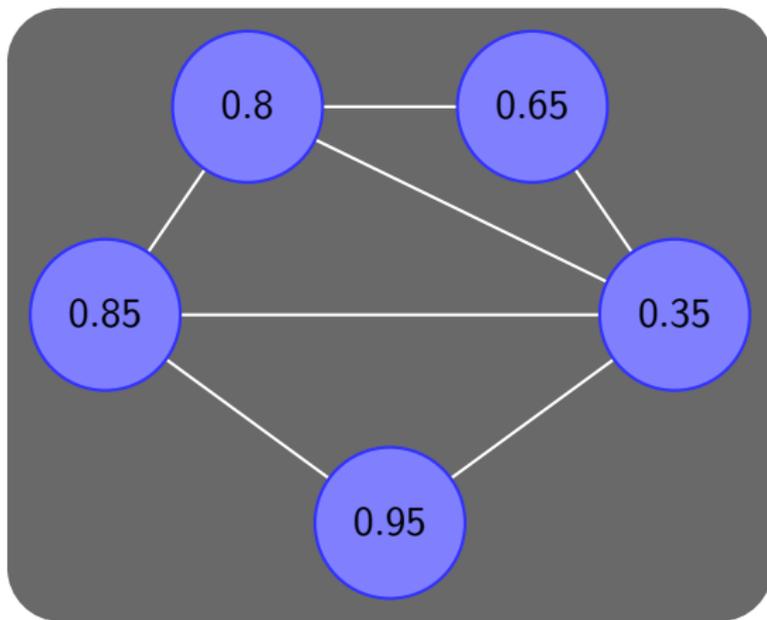
A Simple Initial System Model

- Edges are not power lines: they represent a load sharing arrangement
- Equal sharing of offline generator loads: all graph neighbors take up the offline generator's load
- Underlying electrical network is assumed to be capable of supporting the imposed redundancy

A Simple Failure Model

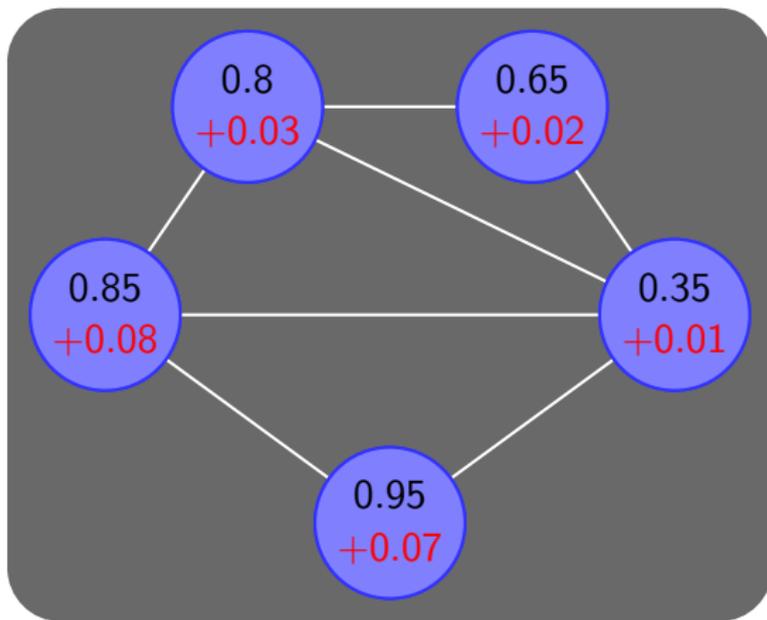
- In steady state, the load demand at each node is below rated capacity
- Initial load is modeled as independent random variables at each node
- Load disturbances increase load at nodes
- Disturbance is modeled as independent random variables at each node
- The load disturbances cause a few generators to go offline
- The offline generator loads get picked up by graph neighbors leading to a propagation of the disturbance and potentially more failures

A Simple Failure Model



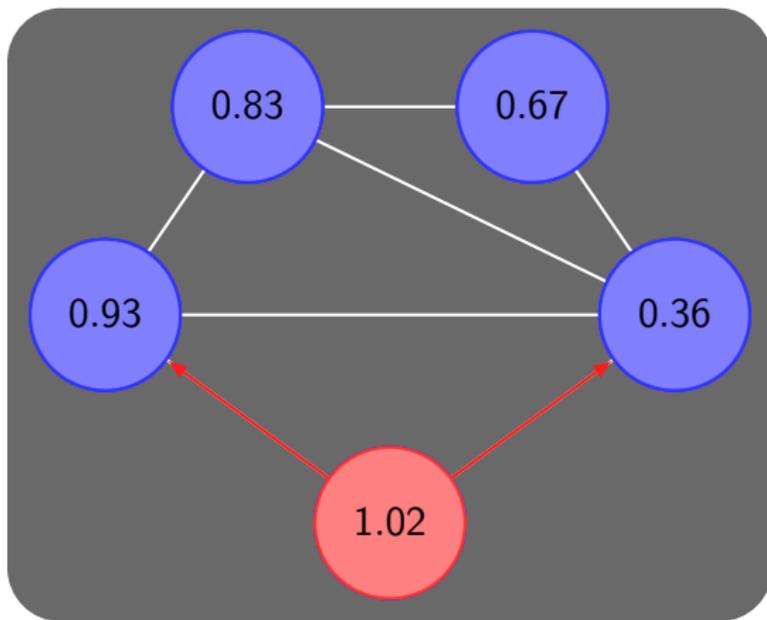
An example of a blackout resulting from a cascade

A Simple Failure Model



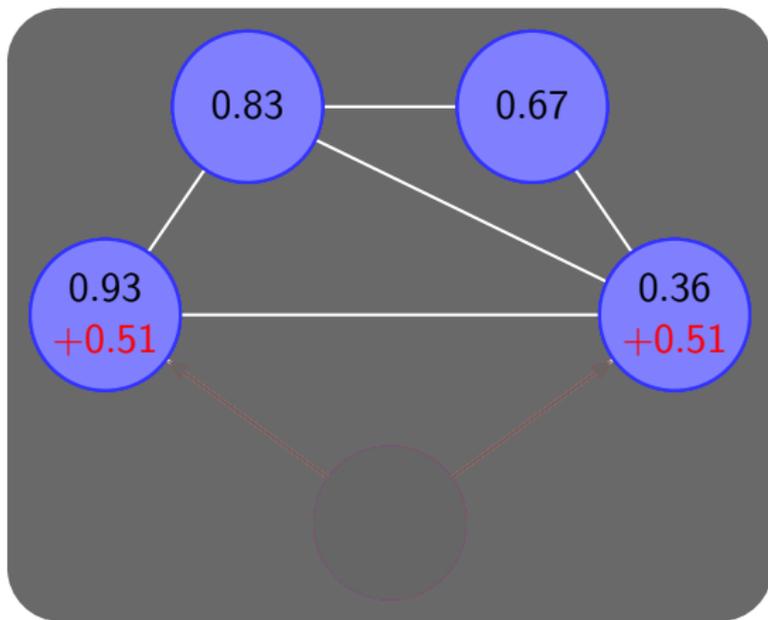
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A Simple Failure Model



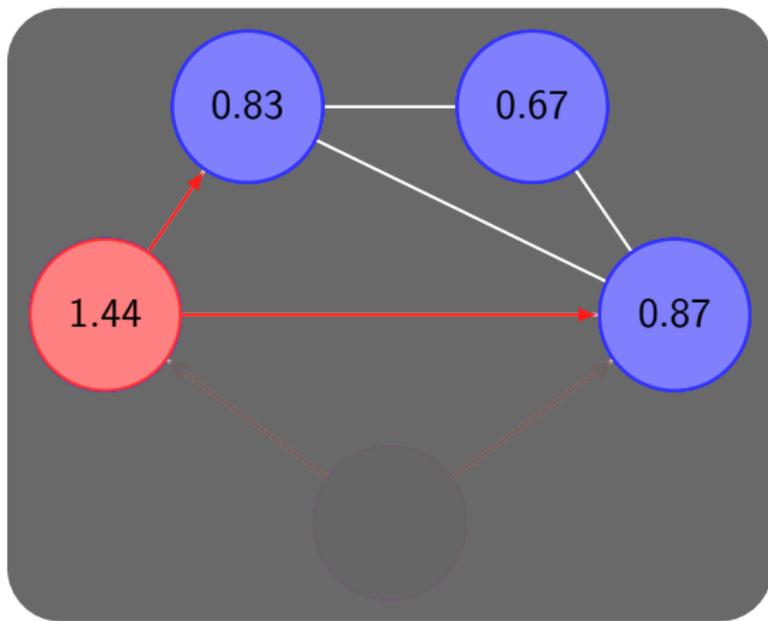
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A Simple Failure Model



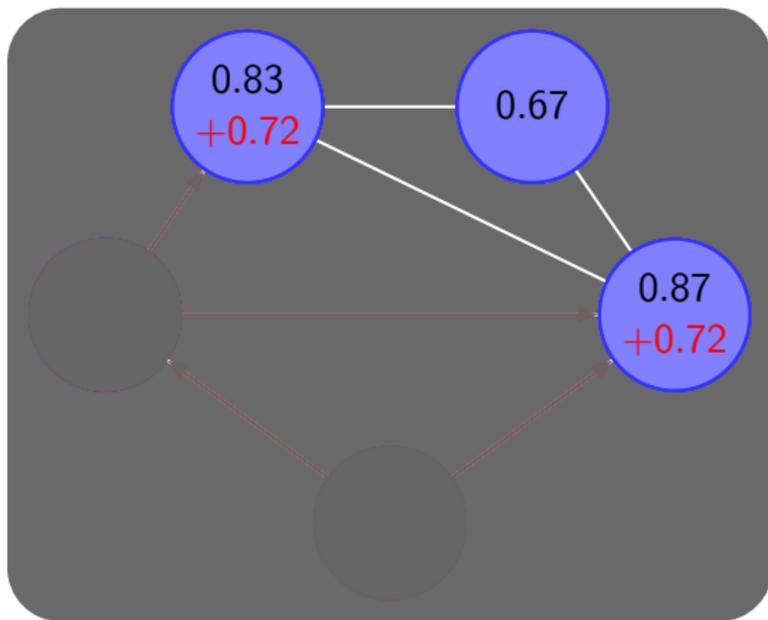
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A Simple Failure Model



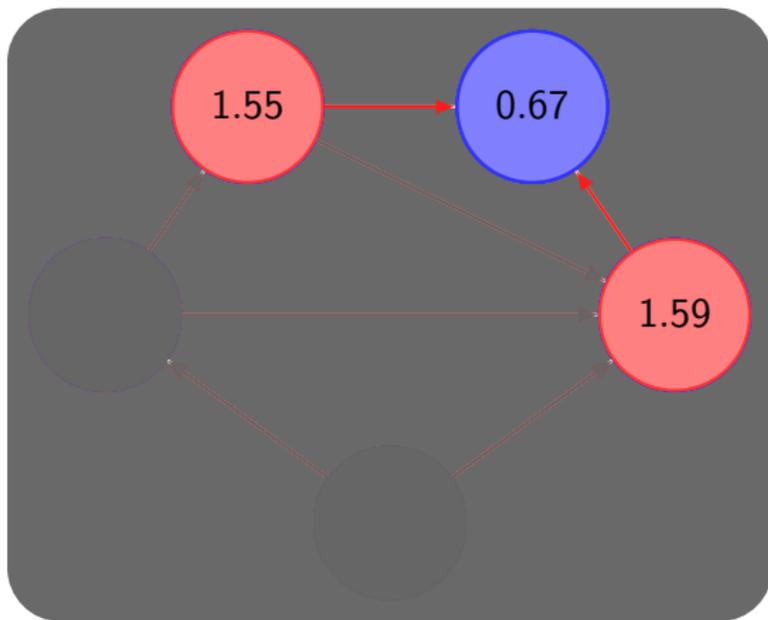
An example of a blackout resulting from a cascade

A Simple Failure Model



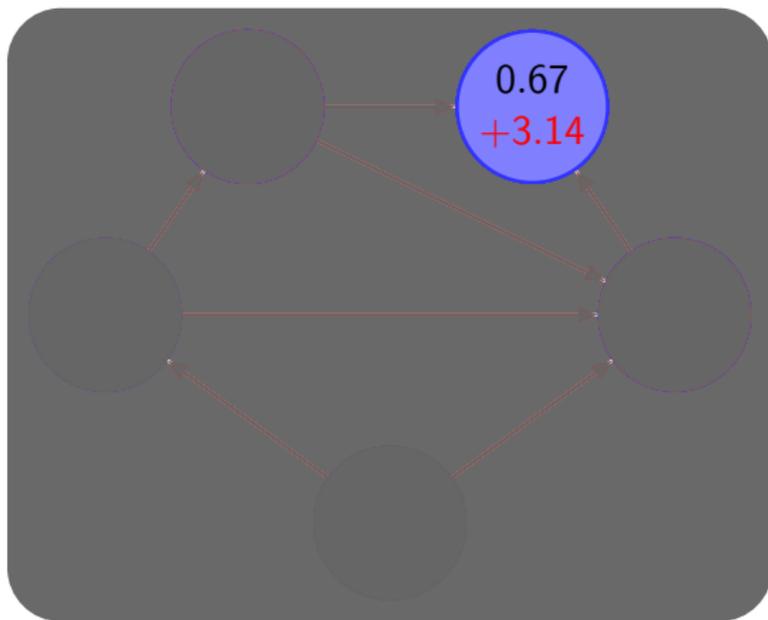
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A Simple Failure Model



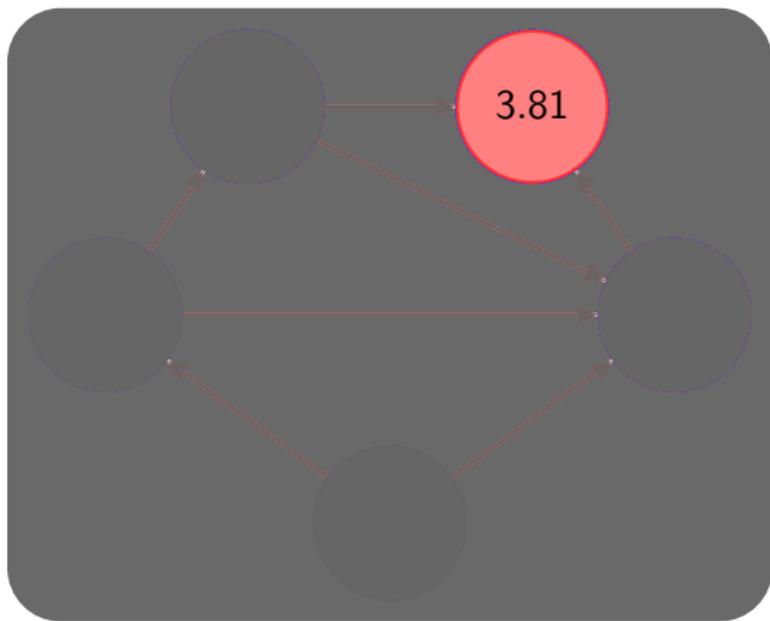
An example of a blackout resulting from a cascade

A Simple Failure Model



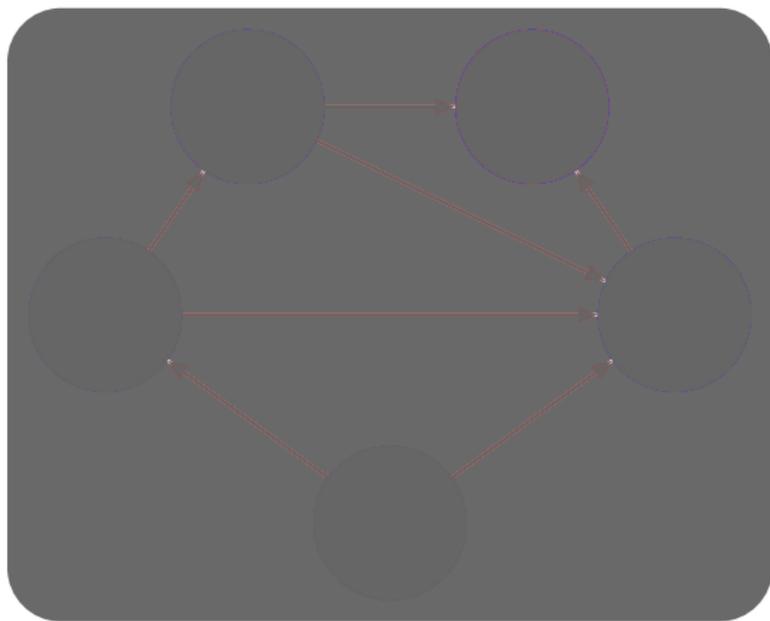
An example of a blackout resulting from a cascade

A Simple Failure Model



An example of a blackout resulting from a cascade

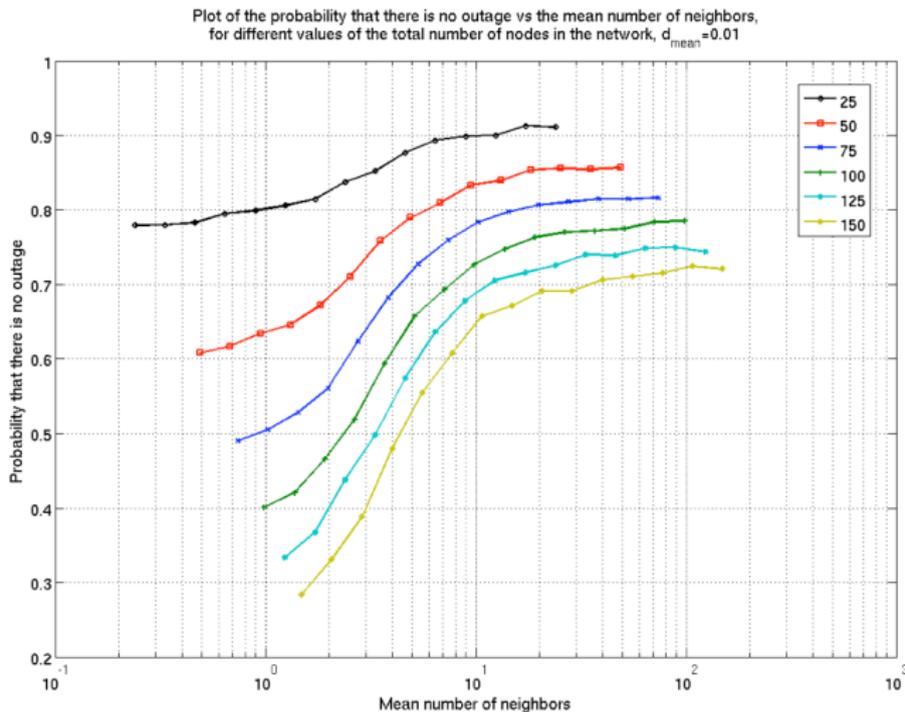
A Simple Failure Model



An example of a blackout resulting from a cascade

Fully Connected Redundancy Graph

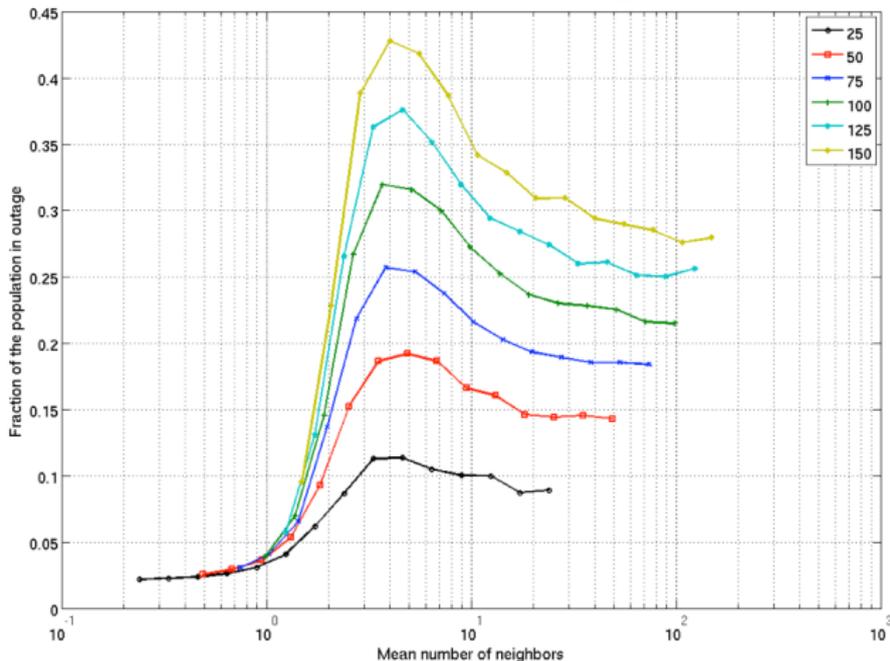
- There is load sharing arrangement between every pair of nodes
- Intuitively - good for robustness
- Good for customers - *outage probability* is least:



Fully Connected Redundancy Graph

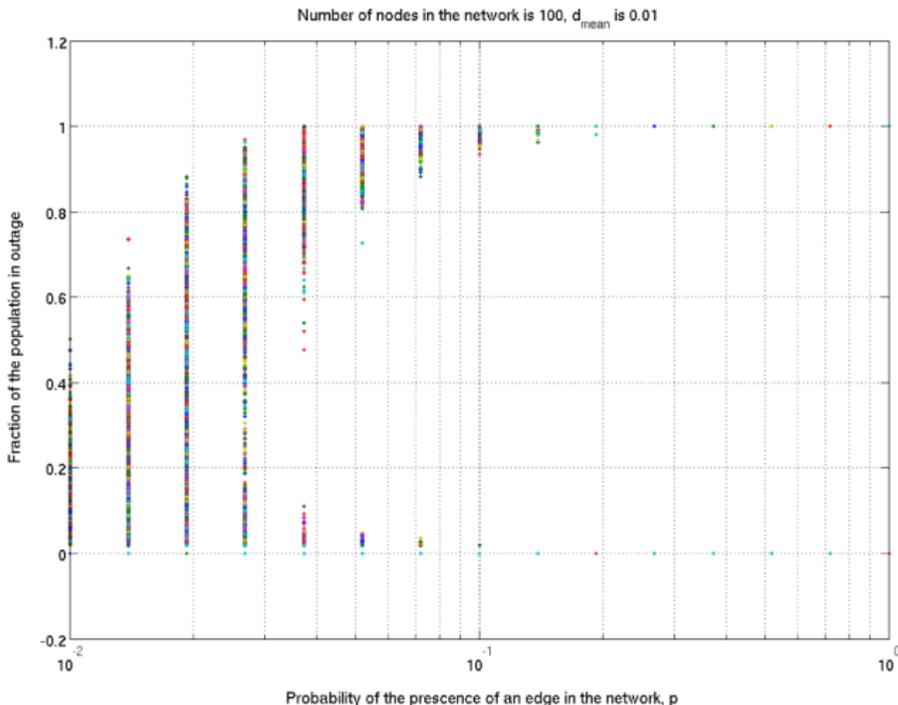
- Not good under other metrics
- Sparse network - cascades subside easily
- Somewhat sparser redundancy graph is good for utilities -
Expected % of population in outage is least:

Plot of the fraction of the population in outage vs the mean number of neighbors, for different values for the nodes in the network, $d_{\text{mean}} = 0.01$



Fully Connected Redundancy Graph

- Fully connected redundancy graph: either there is no outage or there is a total blackout:

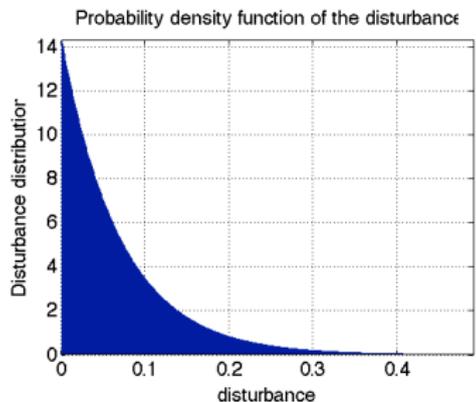
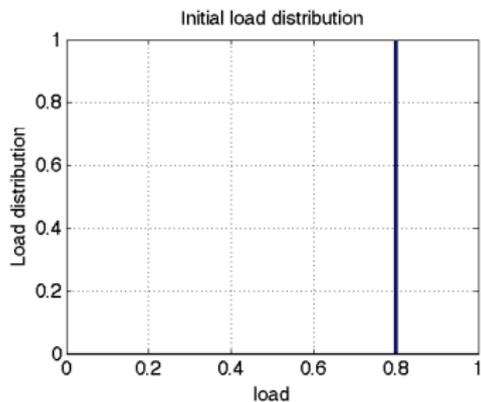


- Fully connected redundancy graph
- Initial loads at each node is a constant
- Disturbance is exponential with mean d_m

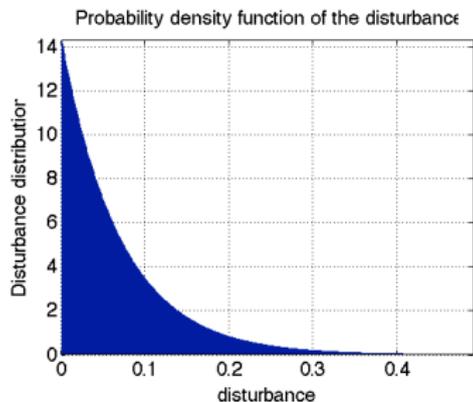
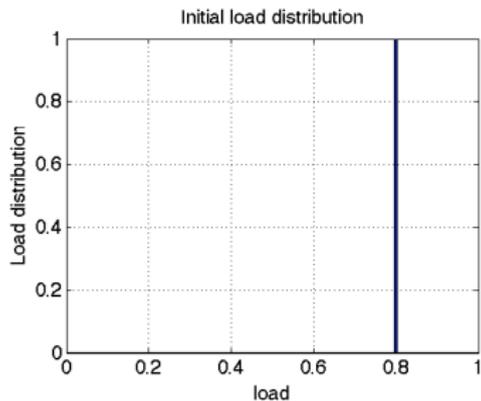
Result

There is a $d_{critical}$ such that, when $d_m < d_{critical}$, cascading failures subside with probability 1, and when $d_m \geq d_{critical}$, all nodes fail with probability 1

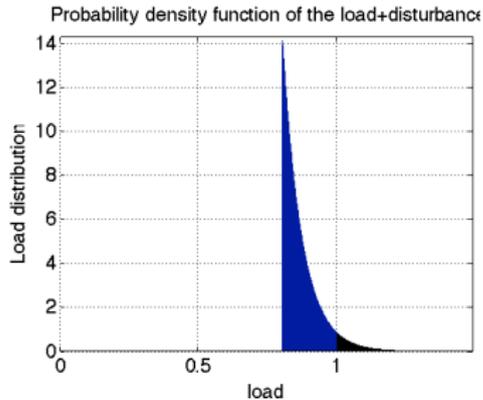
Initial distributions



Initial distributions



Resulting convolution



- a_n = least possible load at the online nodes in stage n
- p_n = probability that a node which is alive at stage n goes offline at stage $(n + 1)$
- \mathcal{D}_n = prob. distribution of the re-distributed load at stage n
- \mathcal{L}_n = prob. distribution of the total load at stage n
- N = total number of nodes
- N_{off} = number of nodes which go offline due to the added disturbance
- Load after re-distribution:

$$L_{j_i}(1) = L_{j_i}(0) + \frac{\sum_{i=0}^{N_{off}} L_{k_i}(0)}{(N - N_{off})}$$

- $N_{off} \sim \text{Binomial}(N, p_0)$
- $\mathcal{L}_0 \sim \delta(a_0)$ and $\mathcal{D}_0 \sim \text{Exponential}(d_m)$
- $\mathcal{D}_n =$ prob. distribution of the re-distributed load at stage n
- $L_{j_i}(0) : f_{L_{j_i}(0)}(x) = f_{\mathcal{L}_0 + \mathcal{D}_0}(x | \mathcal{L}_0 + \mathcal{D}_0 < 1)$
- $L_{k_i}(0) : f_{L_{k_i}(0)}(x) = f_{\mathcal{L}_0 + \mathcal{D}_0}(x | \mathcal{L}_0 + \mathcal{D}_0 \geq 1)$

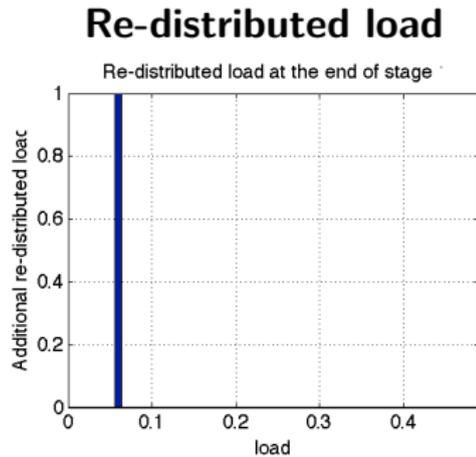
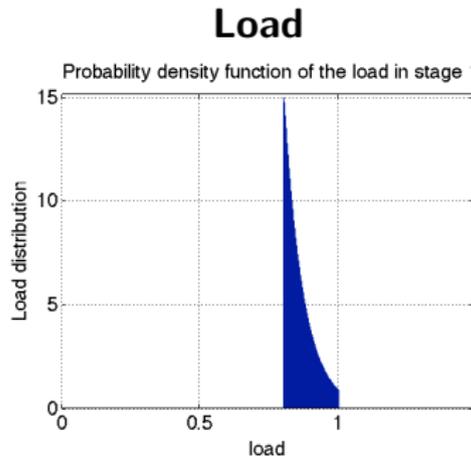
Re-distributed load tends to a constant for large networks

$$\begin{aligned} \lim_{N \rightarrow \infty} S_{N_{off}} &= \lim_{N \rightarrow \infty} \frac{N_{off}/N}{1 - N_{off}/N} \cdot \frac{\sum_{i=0}^{N_{off}} L_{k_i}(0)}{N_{off}} \\ &\stackrel{p}{=} \frac{p_0}{1 - p_0} \cdot \mu_0 \end{aligned}$$

- $\mu_0 = E(L_{k_i}(0)) = \int_{x=1}^{\infty} x \cdot f_{\mathcal{L}_0 + \mathcal{D}_0}(x | \mathcal{L}_0 + \mathcal{D}_0 \geq 1) dx$
- $p_0 = \Pr(\mathcal{L}_0 + \mathcal{D}_0 \geq 1) = \int_{x=1}^{\infty} f_{\mathcal{L}_0 + \mathcal{D}_0}(x) dx$
- **Consequence:** *Loads at all online nodes at each stage are independent and identically distributed (iid)*

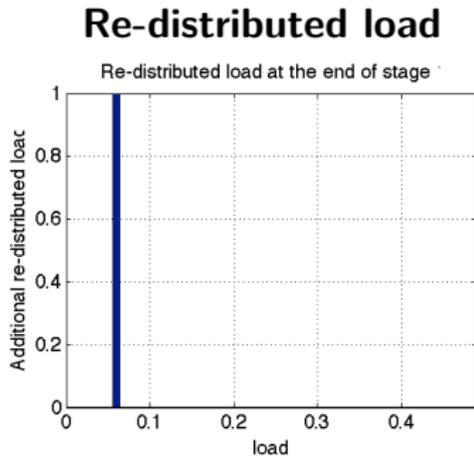
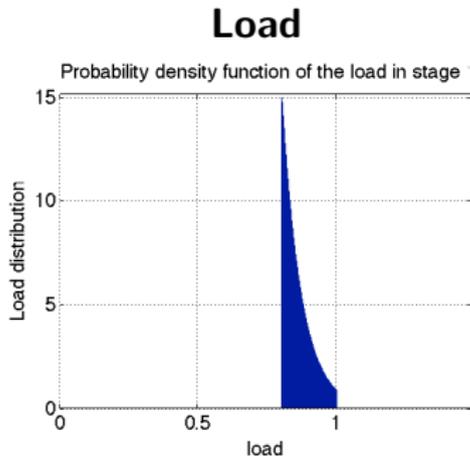
Analysis Proceeds in Stages

Stage 1:

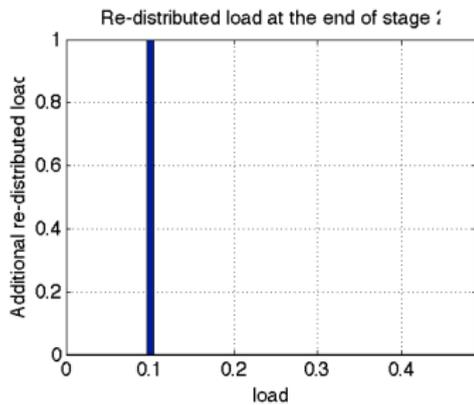
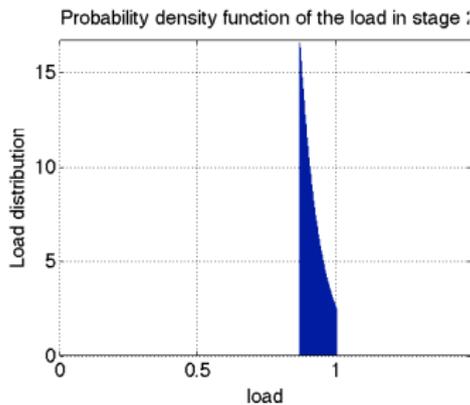


Analysis Proceeds in Stages

Stage 1:



Stage 2:



Recursive Equations Governing System Evolution

- 1 Initialize: $p_0 = e^{-\frac{1-a_0}{d_m}}$, $\mathcal{D}_1 = \frac{p_0}{1-p_0}(1 + d_m)$, $a_1 = a_0$,

$$p_1 = \frac{e^{-\frac{1-a_1}{d_m}}}{1 - e^{-\frac{1-a_1}{d_m}}} \left(e^{\frac{\mathcal{D}_1}{d_m}} - 1 \right)$$

- 2 For $n = (2, \dots, N_{iterations})$, do:

If $((a_{n-1} + \mathcal{D}_{n-1}) > 1)$ and $(a_{n-1} < 1)$, STOP
else:

(a) $a_n = a_{n-1} + \mathcal{D}_{n-1}$

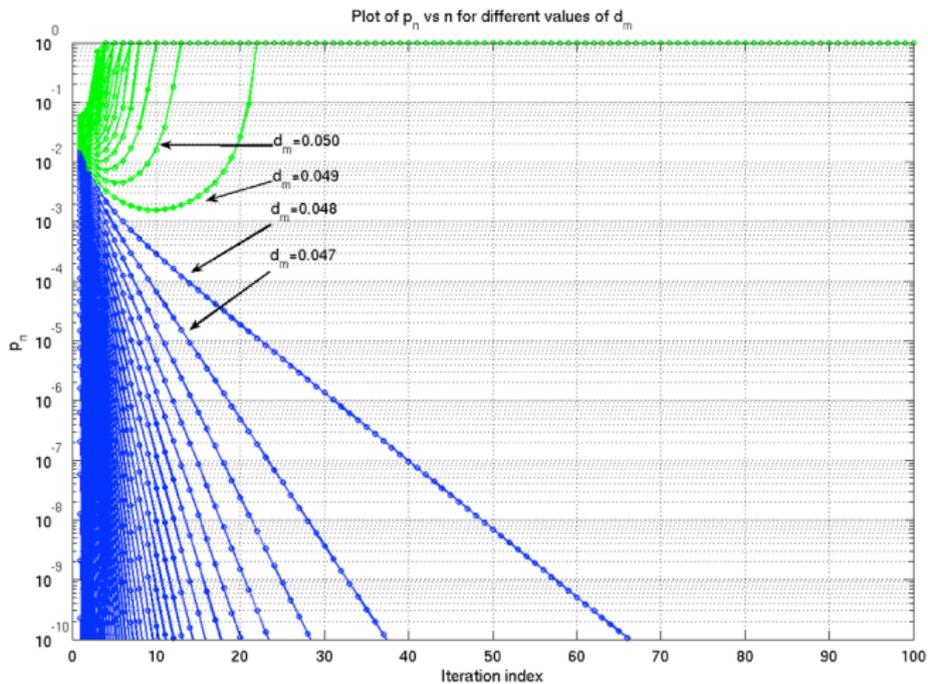
(b) $\mu_{n-1} = 1 + d_m - \frac{\mathcal{D}_{n-1}}{e^{\frac{\mathcal{D}_{n-1}}{d_m}} - 1}$

(c) $\mathcal{D}_n = \frac{p_{n-1}}{1-p_{n-1}} \cdot \mu_{n-1}$

(d) $p_n = \frac{e^{-\frac{1-a_n}{d_m}}}{1 - e^{-\frac{1-a_n}{d_m}}} \left(e^{\frac{\mathcal{D}_n}{d_m}} - 1 \right)$

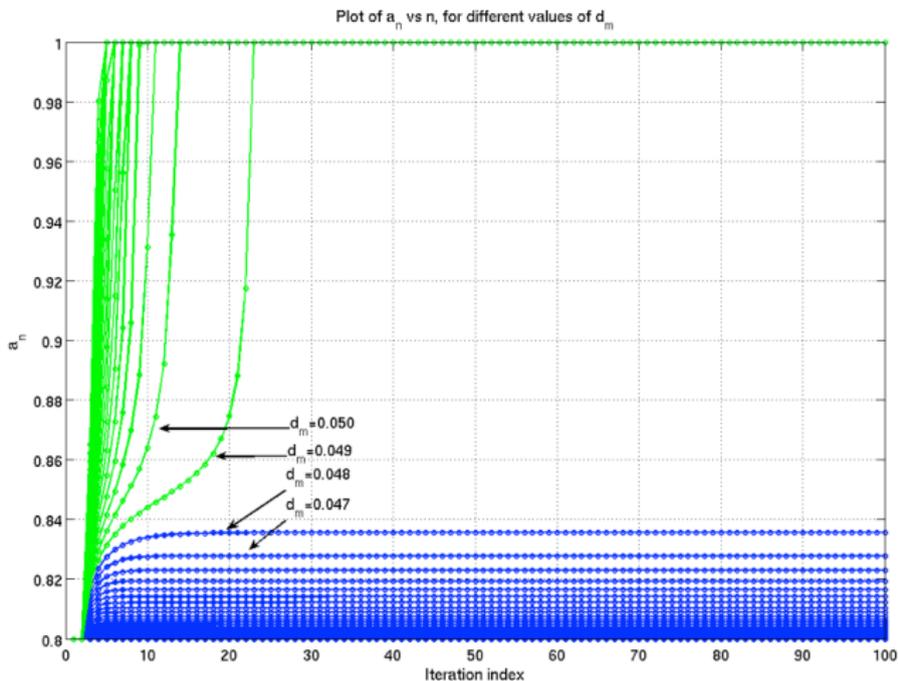
Results: Evolution of p_n

- From the recursive system:



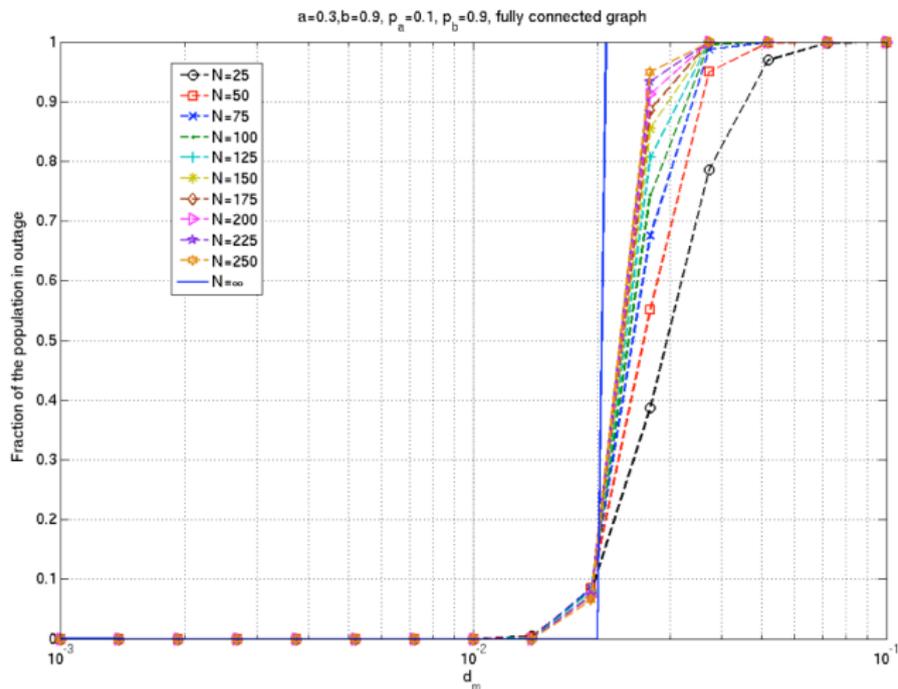
Results: Evolution of a_n

- From the recursive system:



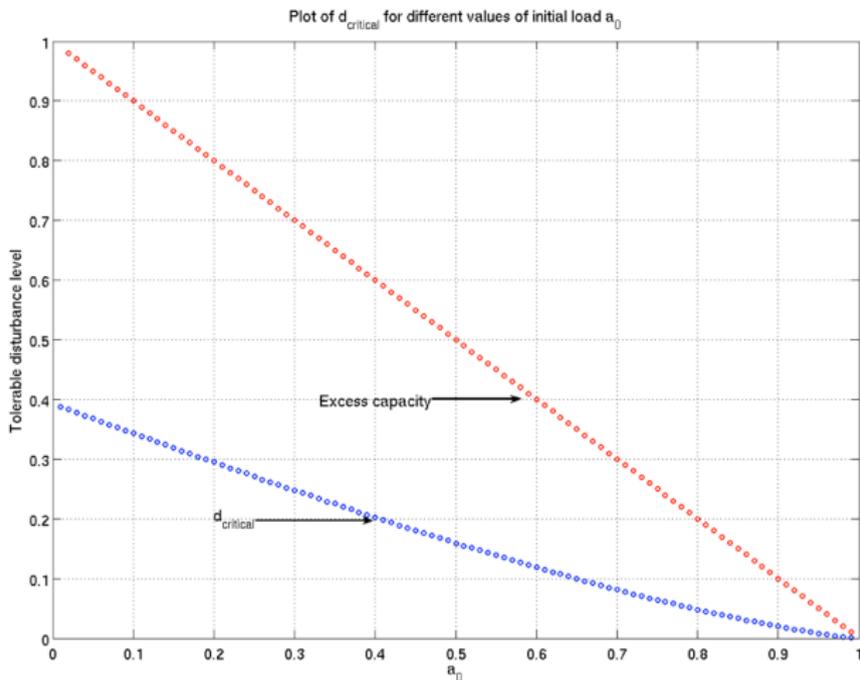
Results: Behavior of a Finite Fully Connected System

- $d_{critical}$ for some large enough finite size networks:



Results: $d_{critical}$ as a function of initial load a_0

- $d_{critical}$ and required excess generation capacity:

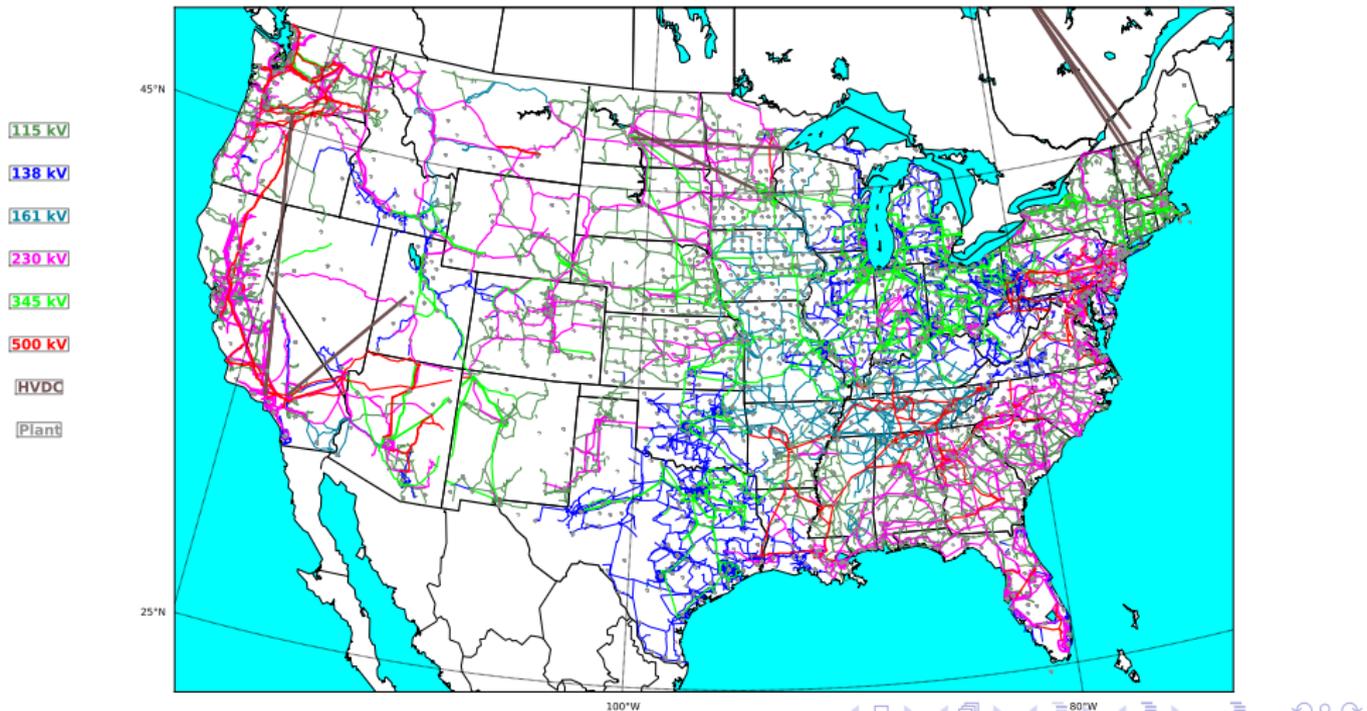


Improvements to the System/Failure Model

- **Generators rarely trip, they are protected using relays etc** - can be modeled using a probability of failure for the protective devices associated with each generator
- **The redundancy graph is never fully connected** - this assumption is used only for simplifying analysis. Simulations can be performed even without this assumption
- **Electricity flow does not behave like this** - when the exact topology of the electrical network is known, simulations can take into account the power flow equations and redistribute load accordingly
- **Load sharing arrangements could change dynamically with market prices** - a market based algorithm can be used to account for this during simulations

Transmission Grid Topology (FEMA, 1993)

- Used ESRI shape-files available from NREL (Originally from FEMA, 1993)
- Used generator data available from EIA, 2008

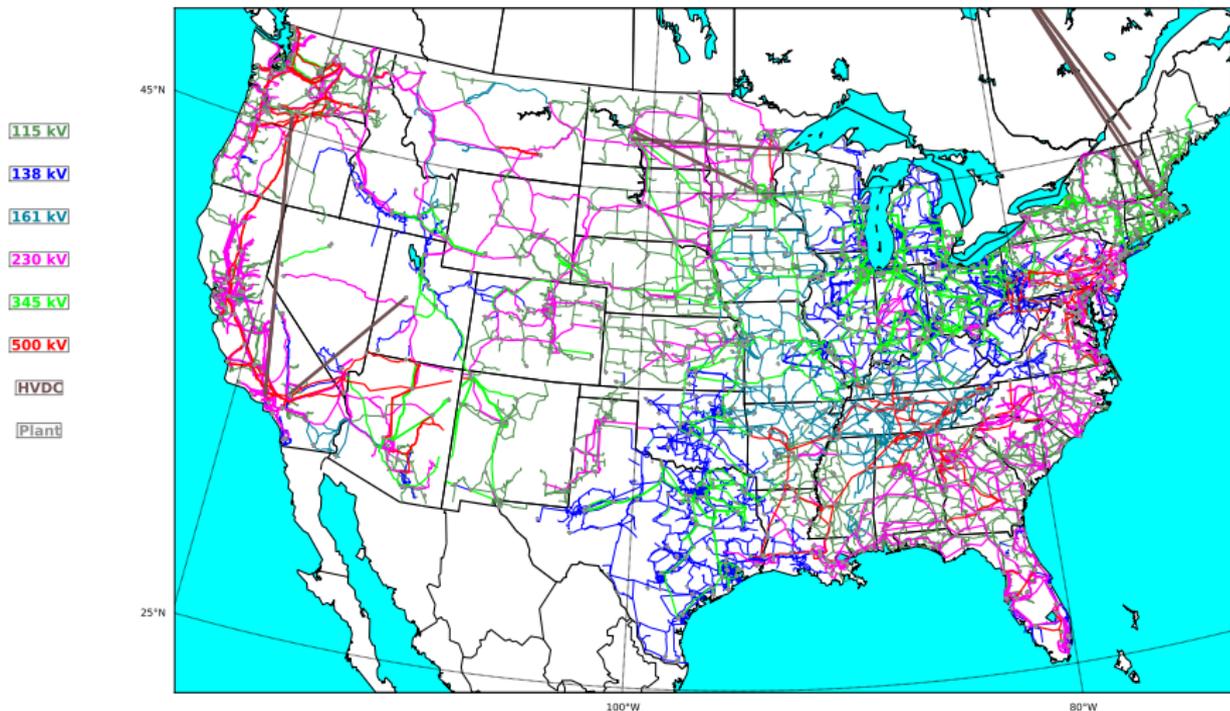


Limitations on Openly Available Data

- Grid topology is somewhat old, but probably not much has changed
- Electrical parameters are not available, but exact length and line voltage are known
- Unit length impedances and transformer parameters may be estimated from similar grids elsewhere
- Generator locations are sometimes available only upto the county level - then we use the county centroid

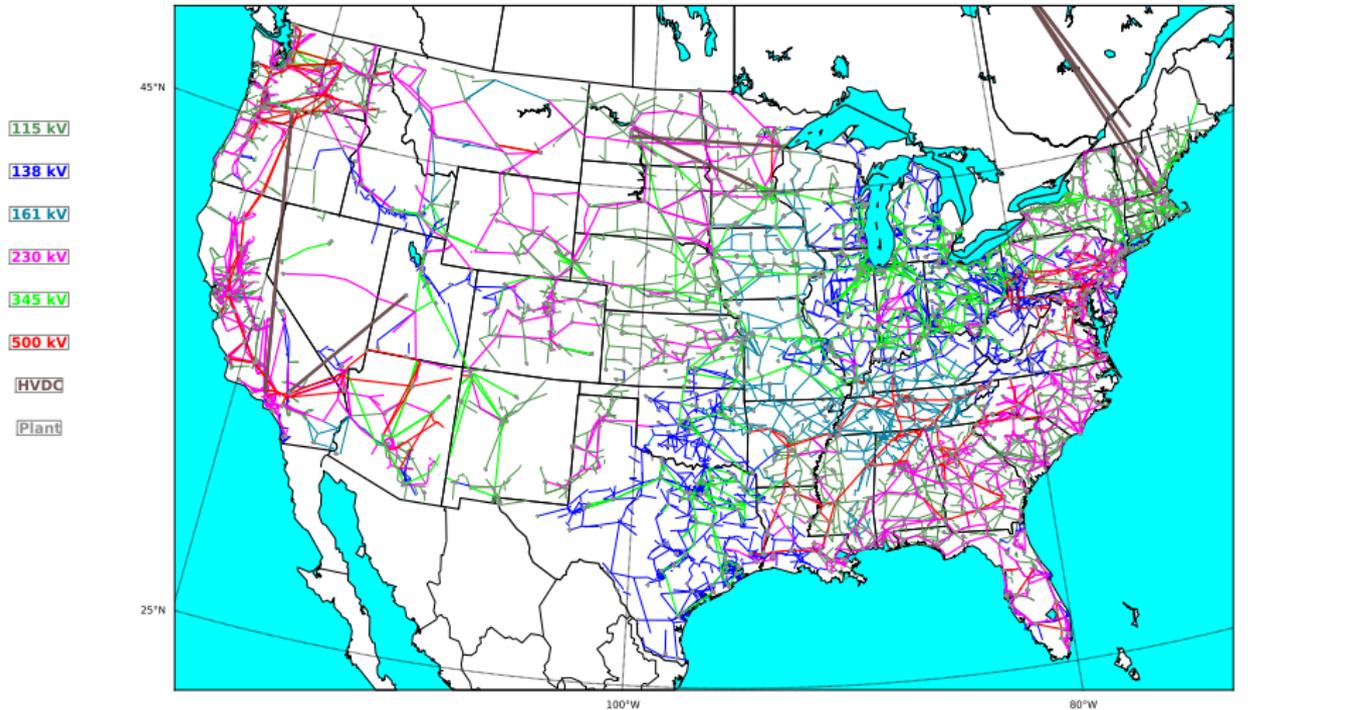
Grid + Generating Plants

- After fixing the generator locations to closest grid point



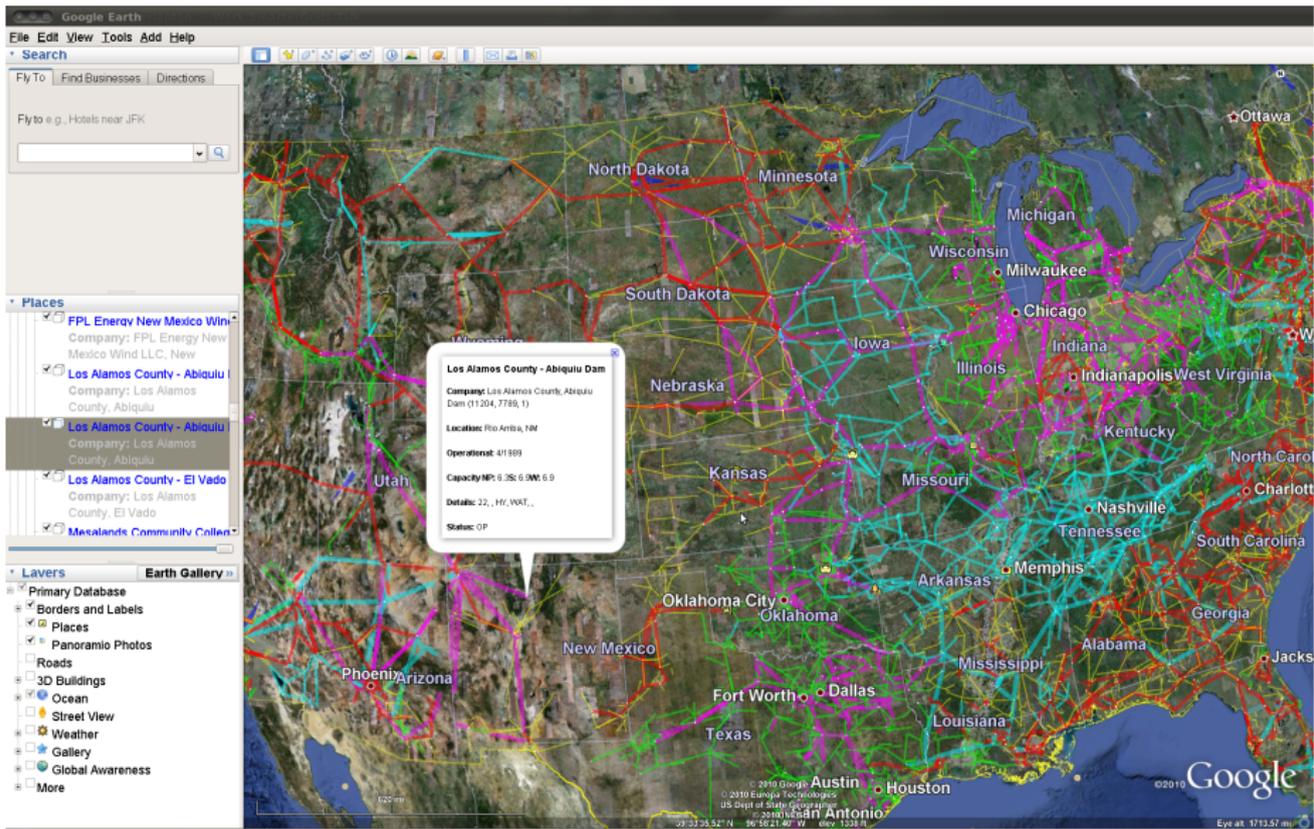
Grid + Generating Plants

- Topology after removing bends (electrically unimportant)



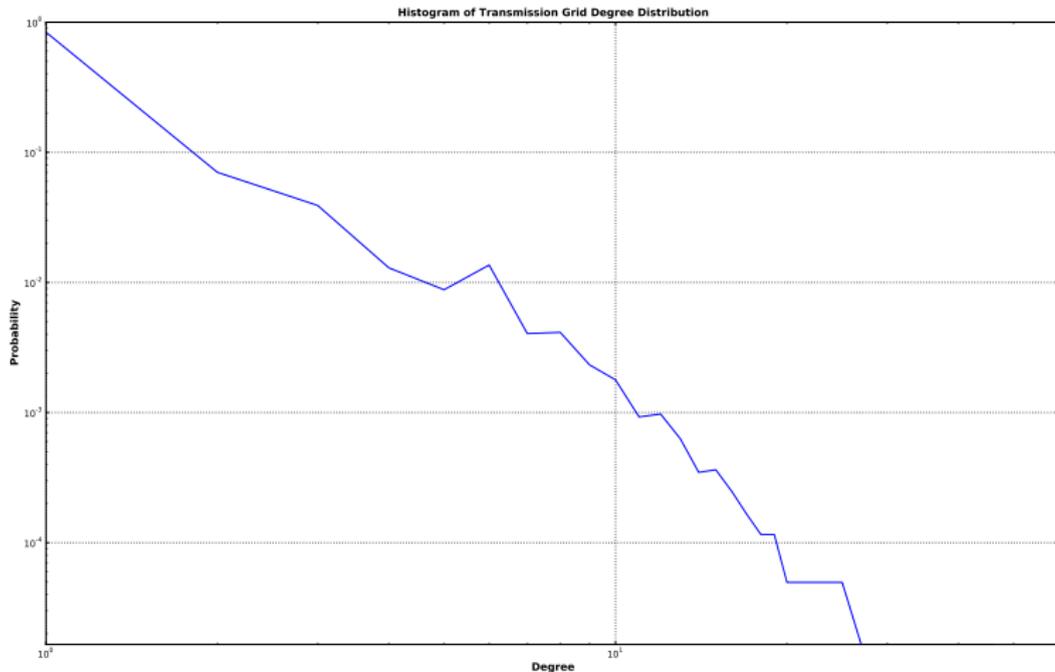
Detail View

Abiquiu



Power Law in Grid Topology

- Transmission grid degree distribution
- Shows an approximate power law behavior:



Conclusions

- Studying cascading failures is important for understanding large scale blackouts
- Developed a simple system and failure model and rigorously analyzed it
- Several improvements are possible to the simple model - however these will make analysis difficult and necessitate simulations
- Further realism is possible only by considering the actual transmission network topology
- Gathered openly available US transmission grid data
- **For future:** simulate more realistic/complicated algorithms for load sharing and failure using the grid data